MATHEMATICS - CET 2022 - VERSION CODE - A -2
Solution

1. If [x] is the greatest integer function not greater than x then $\int_0^8 [x]\,dx$ is equal to

(A) 20 (B) 28 (C) 30 (D) 29

Ans (B)

$$\int_0^8 [x] = 28 = \frac{7(7+1)}{2} = 7\times 4 = 28$$

2. $\int_0^{\pi/2} \sqrt{\sin\theta}\cos^3\theta\, d\theta$ is equal to

(A) $\frac{7}{21}$ (B) $\frac{8}{23}$ (C) $\frac{7}{23}$ (D) $\frac{8}{21}$

Ans (D)

$$I = \int_0^{\pi/2} \sqrt{\sin\theta}\,\cos^2\theta\, d\theta = \frac{8}{21}$$

$$t = \sqrt{\sin\theta} \Rightarrow t^2 = \sin\theta \Rightarrow t^4 = \sin^2\theta$$

$$dt = \frac{1}{2\sqrt{\sin\theta}}\cdot\cos\theta$$

$$I\int_0^1 t\cdot\left(1-t^4\right)\cdot 2t\,dt = \int_0^1 2\left(t^2-t^6\right)dt$$

$$= 2\left[\frac{t^3}{3}-\frac{t^7}{7}\right]_0^1 \qquad = 2\left[\frac{1}{3}-\frac{1}{7}\right] = \frac{8}{21}$$

3. If $e^y + xy = e$ the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at x = 0 is equal to

(A) $\left(\frac{-1}{e}, \frac{1}{e^2}\right)$ (B) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ (C) $\left(\frac{-1}{e}, \frac{-1}{e^2}\right)$ (D) $\left(\frac{1}{e}, \frac{-1}{e^2}\right)$

Ans (A)

$e^y + xy = e$ At $x = 0$, $e^y = e^1 \Rightarrow y = 1$

Point = (0, 1)

$$e^y\frac{dy}{dx} + x\frac{dy}{dx} + y(1) = 0 \text{ At } (0, 1)$$

$$e\frac{dy}{dx} + 0 + 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{1}{e} \qquad \ldots(1)$$

$$ey_2 + \left(-\frac{1}{e}\right)^2 e + 0 + (2)\left(-\frac{1}{e}\right) = 0$$

$$ey_2 = \frac{1}{e} \Rightarrow y_2 = \frac{1}{e^2} \qquad \ldots(2)$$

$$\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right) = \left(-\frac{1}{e}, \frac{1}{e^2}\right)$$

4. The function $f(x)=\log(1+x)-\dfrac{2x}{2+x}$ is increasing on

(A) $(-\infty, 0)$ (B) $(-\infty, \infty)$ (C) $(\infty, -1)$ (D) $(-1, \infty)$

Ans (D)

$$f'(x)=\frac{1}{1+x}-\left[\frac{4}{(2+x)^2}\right]$$

$$f'(x)=\frac{\cancel{4}+x^2+\cancel{4}x-\cancel{4}-\cancel{4}x}{(1+x)(2+x)^2}$$

$$\Rightarrow f'(x)>0 \quad \Rightarrow 1+x>0$$

$$x>-1$$

5. The co-ordinates of the point on the $\sqrt{x}+\sqrt{y}=6$ at which the tangent is equally inclined to the axes is

(A) (6, 6) (B) (4, 4) (C) (1, 1) (D) (9, 9)

Ans (D)

$\dfrac{1}{2\sqrt{x}}+\dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx}=0\Rightarrow\dfrac{dy}{dx}=-\dfrac{\sqrt{y}}{x}$ by verification

6. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

(A) decreasing in $\left(\dfrac{\pi}{2}, \pi\right)$ (B) decreasing in $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

(C) decreasing in $\left[0, \dfrac{\pi}{2}\right]$ (D) increasing in $\left(\pi, \dfrac{3\pi}{2}\right)$

Ans (A)

$f'(x) = 12\sin^2 x\cos x - 12\sin x\cos x + 12\cos x$

$f'(x) = 12\cos x\,[\sin^2 x - \sin x + 1] > 0$ or < 0

$\Rightarrow f'(x) = \cos x > 0$ or < 0

Increasing function in $\left(0, \dfrac{\pi}{2}\right)$ or $\left(3\dfrac{\pi}{2}, \pi\right)$

Decreasing function in $\left(\dfrac{\pi}{2}, \pi\right)$ or $\left(\pi, \dfrac{3\pi}{2}\right)$

7. Area of the region bounded by the curve $y = \tan x$, the x-axis and the line $x=\dfrac{\pi}{3}$ is

(A) $-\log 2$ (B) $\log\dfrac{1}{2}$ (C) $\log 2$ (D) 0

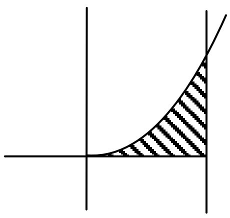
Ans (C)

$y = \tan x$, $y = 0 \Rightarrow x = 0$

$$A=\int_0^{\pi/3}\tan x\,dx$$

$$A=\left[\log(\sec x)\right]_0^{\pi/3}=\log\sec\frac{\pi}{3}-\log\sec 0$$

$$=\log 2$$



8. Evaluate $\int_2^3 x^2 dx$ as the limit of a sum

(A) $\frac{19}{3}$ (B) $\frac{72}{6}$ (C) $\frac{53}{9}$ (D) $\frac{25}{7}$

Ans (A)

$$\int_2^3 x^2 dx = \left(\frac{x^3}{3}\right)_2^3 = \frac{1}{3}[27-8] = \frac{19}{3}$$

9. $\int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$ is equal to

(A) 1 – log 2 (B) log 2 – 1 (C) log 2 (D) – log 2

Ans (A)

$$I = \int_0^{\pi/2} \frac{\cos x \sin x}{1+\sin x} dx$$

t = sinx

$dt = \cos x\, dx$

$$I = \int_0^1 \frac{t}{1+t} dt = \int_0^1 1 - \frac{1}{1+t} dt$$

$$= \left[t - \log(1+t)\right]_0^1$$

$= (1 - \log_2) - (0 - 0) = 1 - \log_e 2$

10. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to

(A) 2(sin x + 2x cos α) + c (B) 2(sin x – x cos α) + c

(C) 2(sin x + x cos α) + c (D) 2(sin x – 2x cos α) + c

Ans (C)

$$\int \frac{\cos x - \cos 2x}{\cos x - \cos \alpha} dx = \int \frac{2\cos^2 x - 1 - \left[2\cos^2 \alpha - 1\right]}{\cos x - \cos \alpha} dx$$

$$= \int \frac{2[\cos x + \cos \alpha][\cos x - \cos \alpha]}{(\cos x - \cos \alpha)} dx$$

$$= 2[\sin x + \cos \alpha x] + c$$

11. $\int_0^1 \frac{xe^x}{(2+x)^3} dx$ is equal to

(A) $\frac{1}{9}.e - \frac{1}{4}$ (B) $\frac{1}{27}.e - \frac{1}{8}$ (C) $\frac{1}{27}.e + \frac{1}{8}$ (D) $\frac{1}{9}.e + \frac{1}{4}$

Ans (A)

$$\int_0^1 \frac{xe^x}{(2+x)^3} dx = \int e^x \left(\frac{1}{(x+2)^2} - \frac{2}{(x+2)^3}\right) dn$$

$$= \left.\frac{e^x}{(x+2)^2}\right|_0^1 = \frac{e}{9} - \frac{1}{4}$$

12. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2| + b\tan^{-1}x + \frac{1}{5}\log|x+2| + c,$ then

(A) $a = \frac{1}{10}\ b = \frac{-2}{5}$ (B) $a = \frac{-1}{10}\ b = \frac{2}{5}$ (C) $a = \frac{1}{10}\ b = \frac{2}{5}$ (D) $a = \frac{-1}{10}\ b = \frac{-2}{5}$

Ans (B)

$$\frac{1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$\Rightarrow 1 = A(x^2+1) + (Bx+C)(x+2)$

$\Rightarrow 1 = (A+B)x^2 + (2B+C)x + (A+2C)$

$\Rightarrow A = -B : 2B = -C : A + 2C = 1$

$\Rightarrow B = -\frac{1}{5} \quad C = \frac{2}{5} \quad 5A = 1$

$$A = \frac{1}{5}$$

$$\int \frac{1}{(x+2)(x^2+1)}dx = \frac{1}{5}\log(x+2) - \frac{1}{10}\log(x^2+1) + \frac{2}{5}\tan^{-1}x + C$$

$\therefore a = -\frac{1}{10} : b = \frac{2}{5}$

13. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and the angle between $\vec{a}$ and $\vec{b}$ is 120°, then the length of the vector $\left|\frac{1^{\vec{a}}}{2} - \frac{1^{\vec{b}}}{3}\right|^2$ is

(A) 1 (B) 2 (C) 3 (D) $\frac{1}{6}$

Ans (C)

$$\left|\frac{\vec{a}}{2} - \frac{\vec{b}}{3}\right|^2 = \frac{|\vec{a}|^2}{4} + \frac{|\vec{b}|^2}{9} - \cancel{2}\frac{|\vec{a}|}{\cancel{2}} \cdot \frac{|\vec{b}|}{3} \cdot \cos 120°$$

$$= 1 + 1 - 2 \cdot \left(\frac{1}{2}\right) = 3$$

Note: Question is printed wrong. GRACE

14. If $|\vec{a} \times \vec{b}| + |\vec{a}.\vec{b}|^2 = 36$ and $|\vec{a}| = 3$ then $|\vec{b}|$ is equal to

(A) 2 (B) 9 (C) 36 (D) 4

Ans

Note: Question is printed wrong. GRACE

15. If $\vec{\alpha} = \hat{i} - 3\hat{j}, \vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$ then $\vec{\beta}_1$ is given by

(A) $\hat{i} + 3\hat{j}$ (B) $\frac{5}{8}(\hat{i} - 3\hat{j})$ (C) $\frac{5}{8}(\hat{i} + 3\hat{j})$ (D) $\hat{i} - 3\hat{j}$

Ans

Note: Incorrect Option. GRACE

16. The sum of the degree and order of the differential equation $(1 + y_1^2)^{2/3} = y_2$ is

(A) 7 (B) 4 (C) 6 (D) 5

Ans (D)

Degree = 3 : order = 2 $\quad \therefore$ sum of degree and order = 5

17. If $\frac{dy}{dx}+\frac{y}{x}=x^2$, then $2y(2)-y(1)=$

(A) $\frac{13}{4}$ (B) $\frac{11}{4}$ (C) $\frac{15}{4}$ (D) $\frac{9}{4}$

Ans (C)

Given, $\frac{dy}{dx}+\frac{1}{x}y=x^2$

$I.F=e^{\int\frac{1}{x}.dx}=e^{\log x}=x$

$y.x=\int x^3.dx+c$

$y.x=\frac{x^4}{4}+c$

$y=\frac{x^3}{4}+\frac{c}{x}$

$y(2)=2+\frac{c}{2}$

$2y(2) = 4 + c$ …(1)

$y(1)=\frac{1}{4}+c$ …(2)

From (1) and (2), $2y(2)-y(1)=4+c-\frac{1}{4}-c$

$2y(2)-y(1)=4-\frac{1}{4}=\frac{15}{4}$

18. The solution of the differential equation $\frac{dy}{dx}=(x+y)^2$ is

(A) $\cot^{-1}(x + y) = x + c$ (B) $\tan^{-1}(x + y) = x + c$

(C) $\tan^{-1}(x + y) = 0$ (D) $\cot^{-1}(x + y) = c$

Ans (B)

$\frac{dy}{dx}=(x+y)^2$

Take $z = x + y$

$\frac{dz}{dx}=1+\frac{dy}{dx}$

$\Rightarrow \frac{dz}{dx}-1=z^2$

$\Rightarrow \frac{dz}{1+z^2}=dx$

$\Rightarrow \tan^{-1} z = x + c$

$\Rightarrow \tan^{-1}(x + y) = x + c$

19. If y(x) be the solution of differential equation $x\log x\frac{dy}{dx}+y=2x\log x$, $y(e)$ is equal to

(A) 2e (B) e (C) 0 (D) 2

Ans

$$\frac{dy}{dx}+\frac{1}{x\log x}\ y=2$$

$$IF=e^{\int\frac{1}{x\log x}dx}=e^{\int\frac{dt}{t}}=e^{\log t}=t=\log x$$

$$y.\log x=\int 2.\log x\ dx+c$$

$$y.\log x=\log x.2x-\int 2x.\frac{1}{x}dx+c$$

$$y.\log x=2x\log x-2x+c$$

$$y(x)=2x-\frac{2x}{\log x}+\frac{c}{\log x}$$

$$\therefore\ y(e)=2e-2e+c=c$$

Note: Incorrect option. GRACE

20. The dietician has to develop a special diet using two foods X and Y. Each packet (containing 30 g) of food. X contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Y contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. The corner points of the feasible region are

(A) (2, 72), (40, 15), (115, 0) (B) (2, 72), (40, 15), (15, 20)

(C) (2, 72), (15, 20), (0, 23) (D) (0, 23), (40, 15), (2, 72)

Ans (C)

	X	Y
Calcium	12	3
Iron	4	20
Cholesterol	6	4
Vitamin A	6	3

Constraints: $12x + 3y \geq 240$

$4x + 20y \geq 460$

$6x + 3y \leq 300$

$12x + 3y = 240$

Put $x = 0 : y = 80$ (0, 80)

$y = 0 : x = 20$ (20, 0)

$4x + 20y = 460$

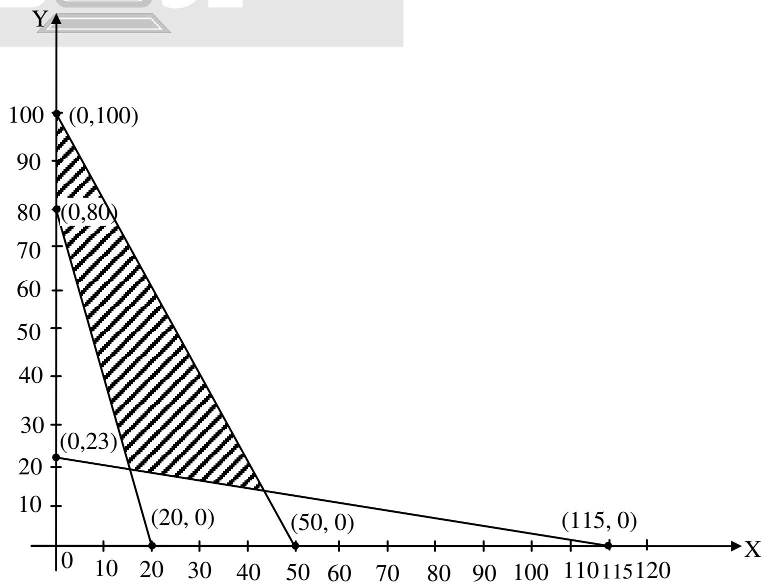
Put $x = 0 : y = 23$ (0, 23)

$y = 0 : x = 115$ (115, 0)

$6x + 3y = 300$

Put $x = 0 : y = 100$ (0, 100)

$y = 0 : x = 50$ (50, 0)



21. The distance of the point whose position vector is $\left(2\hat{i}+\hat{j}-\hat{k}\right)$ from the plane $\vec{r}\cdot\left(\hat{i}-2\hat{j}+4\hat{k}\right)=4$ is

(A) $\frac{-8}{21}$ (B) $\frac{8}{\sqrt{21}}$ (C) $8\sqrt{21}$ (D) $\frac{-8}{\sqrt{21}}$

Ans (B)

A (2, 1, – 1)

Plane: $\vec{r}\cdot\left(\hat{i}-2\hat{j}+4\hat{k}\right)=4$

i.e., x – 2y + 4z – 4 = 0

$$\therefore d=\left|\frac{2-2(1)+4(-1)-4}{\sqrt{1+4+16}}\right|=\left|\frac{-8}{\sqrt{21}}\right|$$

$$=\frac{8}{\sqrt{21}}$$

22. The co-ordinates of foot of the perpendicular drawn from the origin to the plane 2x – 3y + 4z = 29 are

(A) (–2, –3, 4) (B) (2, 3, 4) (C) (2, –3, –4) (D) (2, –3, 4)

Ans (D)

2x – 3y + 4z = 29

$\therefore \sqrt{4+9+16}=\sqrt{29}$ i.e., $\frac{2}{\sqrt{29}}x-\frac{3}{\sqrt{29}}y+\frac{4}{\sqrt{29}}z=\frac{29}{\sqrt{29}}$

$$(ld,\ md,\ nd)=\left(\frac{2}{\sqrt{29}}\sqrt{29},\frac{-3}{\sqrt{29}}\times\frac{4}{\sqrt{29}}\times\sqrt{29}\right)$$

= (2, – 3, 4)

23. The angle between the pair of lines $\frac{x+3}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{1}=\frac{y-4}{4}=\frac{z-5}{2}$ is

(A) $\theta=\cos^{-1}\left[\frac{5\sqrt{3}}{16}\right]$ (B) $\theta=\cos^{-1}\left[\frac{27}{5}\right]$ (C) $\theta=\cos^{-1}\left[\frac{8\sqrt{3}}{15}\right]$ (D) $\theta=\cos^{-1}\left[\frac{19}{21}\right]$

Ans

$$\cos\theta=\frac{3.1+5.4+4.2}{\sqrt{9+25+16}\ \sqrt{1+16+4}}$$

$$\cos\theta=\frac{31}{5\sqrt{2}\sqrt{21}}$$

Note: Incorrect option. GRACE

24. The corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5), then the minimum value of z = 4x + 6y occurs at

(A) only two points (B) finite number of points

(C) infinite number of points (D) only one point

Ans (C)

z = 4x + 6y

$$\text{At}\ \left.\begin{matrix}(0\cdot 2) & z=12\\ (3\cdot 0) & z=12\end{matrix}\right\}\text{multiple min}$$

(6·0) z = 24

(6·8) z = 72 max

(0·5) z = 30

Hence, F_{min} occurs at all points on the line joining (0·2) and (3·0).

25. If A and B are two independent events such that $P(\bar{A}) = 0.75$, $P(A \cup B) = 0.65$, and P(B) = x, then find the value of x:

(A) $\frac{7}{15}$ (B) $\frac{5}{14}$ (C) $\frac{8}{15}$ (D) $\frac{9}{14}$

Ans (C)

$P(A') = 0.75 \Rightarrow P(A) = 1 - 0.75 = 0.25$

$P(A \cup B) = 0.65$

$P(A \cup B) = P(A) + P(B) - \frac{P(A \cap B)}{1}$

$= 0.25 + x - P(A)\ P(B)$

$0.65 - 0.25 = x - 0.25\ x$

$0.40 = x\ [0.75\}$

$\frac{0.40}{0.75} = x$

$x = \frac{4/10}{3/4} = \frac{16}{30} = \frac{8}{15}$

26. Find the mean number of heads in three tosses of a fair coin:

(A) 3.5 (B) 1.5 (C) 4.5 (D) 2.5

Ans (B)

$S = \begin{Bmatrix} HHH, \overline{HHT}, \overline{HTH}, HTT, \\ \overline{THH}, THT, TTH, TTT \end{Bmatrix}$

P.D. is

X	0	1	2	3
P(x)	1/8	3/8	3/8	1/8

Mean $= \sum x_i p_i$

$= 0 + \frac{3}{8} + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right)$

$= \frac{3+6+3}{8} = \frac{12}{8} = \frac{3}{2} = 1.5$

27. If A and B are two events such that $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$ and $P(A|B) = \frac{1}{4}$, then $P(A' \cap B')$ is

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{3}{16}$ (D) $\frac{1}{12}$

Ans (B)

$P(A) = \frac{1}{2} : P(B) = \frac{1}{3} : P(A/B) = \frac{1}{4}$

$\frac{P(A \cap B)}{P(B)} = \frac{1}{4}$

$\Rightarrow P(A \cap B) = \frac{1}{3} \times \frac{1}{4}$

$P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$$=1-\left\{\frac{1}{2}+\frac{1}{3}-\frac{1}{12}\right\}$$

$$=1-\left\{\frac{6+4-1}{12}\right\} \quad =1-\frac{9}{12}=\frac{3}{12}=\frac{1}{4}$$

28. A pandemic has been spreading all over the world. The probabilities are 0.7 that there will be a lockdown, 0.8 that the pandemic is controlled in one month if there is a lockdown and 0.3 that it is controlled in one month if there is no lockdown. The probability that the pandemic will be controlled in one month is

(A) 0.46 (B) 0.65 (C) 1.65 (D) 1.46

Ans (B)

$P_{|E_1|}=0.7$ (Lockdown)

$P_{|E_2|}=0.3$ (No lockdown)

A – controlled in one month

$P(A/E_1) = 0.8$

$P(A/E_2) = 0.3$

$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2)$ P;(A/E_2)

$= (0.7)\ (0.8) + (0.3)\ (0.3)$

$= 0.56 + 0.09$

$= 0.65$

29. The degree measure of $\frac{\pi}{32}$ is equal to

(A) 4° 30′ 30″ (B) 5° 30′ 20″ (C) 5° 37′ 20″ (D) 5° 37′ 30″

Ans (D)

$$\frac{\pi}{32}=\frac{\pi}{32}\times 1C$$

$$=\frac{90}{16}=\frac{45}{8}=5°37'30"$$

30. The value of $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$ is

(A) $\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{1}{2}$

Ans (A)

$$\sin\frac{5\pi/12}{A}\sin\frac{\pi/12}{B}=-\frac{1}{2}\left[\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)\right]$$

$$=-\frac{1}{2}\left[\cos\frac{\pi}{2}-\cos\frac{\pi}{3}\right]$$

$$-\frac{1}{2}\left[0-\frac{1}{2}\right]=\frac{1}{4}$$

31. $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=$

(A) $2\cos\frac{\theta}{2}$ (B) sin 2 θ (C) 2 cos θ (D) 2 sin θ

Ans (C)

$\sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$

$= \sqrt{2+\sqrt{2+\sqrt{2(2\cos^2 4\theta)}}}$

$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$= \sqrt{2+\sqrt{2(2\cos^2 2\theta)}}$

$= \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2(2\cos^2 \theta)}$

$= 2\cos\theta$

32. If A = {1, 2, 3, …. 10} then number of subsets of A containing only odd numbers is

(A) 30 (B) 31 (C) 27 (D) 32

Ans (B)

A = {1, 2, 3,...10}

Only odd numbers

$2^5 - 1 = 32 - 1 = 31$

33. Suppose that the number of elements in set A is p, the number of elements in set B is q and the number of elements in $A \times B$ is 7 then $p^2 + q^2 =$

(A) 49 (B) 50 (C) 51 (D) 42

Ans (B)

n (A) = p

n (B) = q

$n(A \times B) = 7$

pq = 7

$pq = 7 \times 1$

p = 7, q = 1

$p^2 + q^2 = 7^2 + 1^2$

$= 50$

34. The domain of the function $f(x) = \dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

(A) $[-2, 0) \cup (0, 1)$ (B) $[-2, 0) \cap (0, 1)$ (C) $[-2, 1)$ (D) $[-2, 0)$

Ans (A)

$\dfrac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

$x + 2 \geq 0$ $\quad 1 - x > 0$

$x \geq -2$ $\quad 1 > x$

$x < 1$ and $x \neq 0$

$[-2, 0) \cup (0, 1)$

35. The trigonometric function y = tan x in the II quadrant

(A) increases from $-\infty$ to 0 (B) decreases from 0 to ∞

(C) decreases from $-\infty$ to 0 (D) increases from 0 to ∞

Ans (A)

36. The octant in which the point (2, –4, –7) lies is

(A) Fifth (B) Eighth (C) Third (D) Fourth

Ans (B)

37. If $f(x)=\begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3 \end{cases}$, the quadratic equation whose roots are $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$ is

(A) $x^2 - 7x + 8 = 0$ (B) $x^2 - 14x + 49 = 0$ (C) $x^2 - 10x + 21 = 0$ (D) $x^2 - 6x + 9 = 0$

Ans (C)

$$f(x)=\begin{cases} x^2-1, & 0<x<2 \\ 2x+3, & 2\le x<3 \end{cases}$$

$$a = \lim_{x\to 2^-}\left(x^2-1\right) = 2^2-1=3$$

$$b = \lim_{x\to 2^+}\left(2x+3\right) = 2(2)+3$$

$$= 7$$

$x^2 - (a + b)x + ab = 0$

$\Rightarrow x^2 - 10x + 21 = 0$

38. If 3x + i (4x – y) = 6 – i where x and y are real numbers, then the values of x and y are respectively,

(A) 3, 4 (B) 3, 9 (C) 2, 4 (D) 2, 9

Ans (D)

3x + i(4x – y) = 6 – i

3x = 6, 4x – y = –1

x = 2, 4 (2) – y = –1

y = 9

39. If all permutations of the letters of the word MASK are arranged in the order as in dictionary with or without meaning, which one of the following is 19th word?

(A) AMSK (B) KAMS (C) SAMK (D) AKMS

Ans

MASK

AKMS = 3 ! = 6

KAMS = 3! = 6

MAKS = 3! = 6

19th word is SAKM

Note: Incorrect option. GRACE

40. If $a_1, a_2, a_3, \ldots, a_{10}$ is geometric progression and $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

(A) $2(5^2)$ (B) $3(5^2)$ (C) 5^4 (D) 5^3

Ans (C)

$$\frac{a_3}{a_1} = 25$$

$$\frac{ar^2}{a} = 25$$

$$r^2 = 25$$

$$r = 5$$

$$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = 5^4$$

41. If the straight line $2x - 3y + 17 = 0$ is perpendicular to the line passing through the points (7, 17) and (15, β), then β equals

(A) –29 (B) –5 (C) 5 (D) 29

Ans (C)

Perpendicular line is $3x + 2y + k = 0$

At (7, 17) $\Rightarrow 7(3) + 2(17) + k = 0$

$\Rightarrow 21 + 34 + k$

$\Rightarrow k = -55$

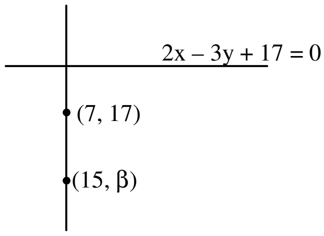
Equation of the line $3x + 2y - 55 = 0$

$(15, \beta) \Rightarrow 3\,(15) + 2\beta - 55 = 0$

$2\beta = 55 - 45$

$2\beta = 10$

$\beta = 5$



42. Let the relation R is defined in N by a R b, if $3a + 2b = 27$ then R is

(A) {(2, 1) (9, 3) (6, 5) (3, 7)} (B) {(1, 12) (3, 9) (5, 6) (7, 3)}

(C) $\left\{\left(0, \frac{27}{2}\right)\right.$ (1, 12) (3, 9) (5, 6) (7, 3)} (D) {(1, 12) (3, 9) (5, 6) (7, 3) (9, 0)}

Ans (B)

$R = \{(a, b) : 3a + 2b = 27\}$

$\{(1, 12)\ (3, 9)\ (5, 6)\ (7, 3)\} \in R$

43. $\lim\limits_{y\to 0} \dfrac{\sqrt{3+y^3} - \sqrt{3}}{y^3} =$

(A) $3\sqrt{2}$ (B) $\dfrac{1}{2\sqrt{3}}$ (C) $\dfrac{1}{3\sqrt{2}}$ (D) $2\sqrt{3}$

Ans (B)

$$\lim_{y\to 0} \frac{\sqrt{3+y^3} - \sqrt{3}}{y^3} \times \frac{\sqrt{3+y^3} + \sqrt{3}}{\sqrt{2+y^3} + \sqrt{3}}$$

$$\lim_{y\to 0} \left(\frac{\left(\cancel{3} + y^3\right) - \cancel{3}}{y^3\sqrt{3+y^3} + \sqrt{3}} \right)$$

$$\lim_{y\to 0} \left(\frac{\cancel{y^3}}{\cancel{y^3}\,\sqrt{3+y^3} + \sqrt{3}} \right) = \frac{1}{2\sqrt{3}}$$

44. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(A) $2\sqrt{6}$ (B) $4\sqrt{\frac{5}{3}}$ (C) $\sqrt{6}$ (D) $2\sqrt{\frac{10}{3}}$

Ans (A)

$$SD=\sqrt{\frac{1}{n}\sum x_p^2-(\overline{x})^2}$$

$$\overline{x}=\frac{k}{4}$$

$$\sqrt{5}=\sqrt{\frac{1}{4}(1+0+1+k^2)-\frac{k^2}{16}}$$

$$5=\frac{2+k^2}{4}-\frac{k^2}{16}$$

$80 = 8 + 4k^2 - k^2$

$3k^2 = 72$

$k^2 = 24$

$k=\pm 2\sqrt{6}$

45. If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is

(A) 8! (B) 0 (C) $8P_7$ (D) 7!

Ans (B)

46. If f : R → R be defined by $f(x)=\begin{cases}2x & : x>3\\ x^2 & : 1<x\le 3\\ 3x & : x\le 1\end{cases}$ then f (–1) + f (2) + f (4) is

(A) 14 (B) 5 (C) 10 (D) 9

Ans (D)

f(–1) + f(2) + f(4) = – 3 + 4 + 8

= 9

47. If $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ then $(aI + bA)^n$ is (where I is the identity matrix of order 2)

(A) $a^nI + b^nA$ (B) $a^2I + a^{n-1}\, b \cdot A$ (C) $a^nI + n \cdot a^{n-1}\, b \cdot A$ (D) $a^n I + n\, a^n\, b\, A$

Ans (C)

By Mathematical induction.

48. If A is a 3 × 3 matrix such that |5 · adj A| = 5 then |A| is equal to

(A) ±5 (B) ±1 (C) ±1/25 (D) ±1/5

Ans (D)

| 5 adj A | = 5

5^3 | adj A | = 5

125 | adj A | = 5

$$|\text{adj } A|=\frac{5}{125}=\sqrt{\frac{1}{25}}$$

$|A|^{n-1} = \frac{1}{25}$

$|A|^2 = \frac{1}{25} \quad \Rightarrow |A| = \frac{1}{5}$

49. If there are two values of 'a' which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

Then the sum of these numbers is

(A) 5 (B) –4 (C) 9 (D) 4

Ans (B)

$\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$1(2a^2 + 4) + 2(4a - 0) + 5(8 - 0) = 86$

$2a^2 + 4 + 8a + 40 = 86$

$2a^2 + 8a - 42 = 0$

$a^2 + 4a - 21 = 0$

$a^2 + 7a - 3a - 21 = 0$

$a(a + 7) - 3(a + 7) = 0$

$a = 3 \qquad a = -7$

Sum of the roots = 3 – 7 = – 4

50. If the vertices of a triangle are (–2, 6) (3, –6) and (1, 5), then the area of the triangle is

(A) 35 sq. units (B) 40 sq. units (C) 15.5 sq. units (D) 30 sq. units

Ans (C)

$\text{Area} = \frac{1}{2}\begin{vmatrix} -2 & 6 & 1 \\ 3 & -6 & 1 \\ 1 & 5 & 1 \end{vmatrix}$

$= \frac{1}{2}(-2(-6-5) - 6(3-1) + 1(15+6)$

$= \frac{1}{2}(22 - 12 + 21)$

$= \frac{1}{2}(31)$

= 15.5 sq. units

51. Domain of $\cos^{-1}[x]$ is, where [ ] denotes a greatest integer function

(A) [–1, 2) (B) (–1, 2] (C) (–1, 2) (D) [–1, 2]

Ans (A)

Domain of $\cos^{-1} t$ is $t \in [-1, 1]$

$\Rightarrow [x] \in [-1, 1]$

From the graph of

[x],

Domain of $\cos^{-1}[x]$ is $x \in [-1, 2)$

52. If A is a matrix of order 3×3, then $(A^2)^{-1}$ is equal to

(A) $(-A)^{-2}$ (B) $(-A^2)^2$ (C) $(A^{-1})^2$ (D) A^2

Ans (C)

$$(A^2)^{-1} = (A \cdot A)^{-1}$$
$$= A^{-1} \cdot A^{-1}$$
$$= (A^{-1})^2$$

53. If $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$, then the inverse of the matrix A^3 is

(A) –A (B) A (C) –1 (D) 1

Ans (B)

$$A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore\ A^3 = A \cdot A^2 = A \cdot I = A$

Inverse of A^3 = Inverse of A $= \frac{1}{|A|}$ adj A

$$= -\frac{1}{1}\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ +3 & -2 \end{bmatrix}$$

$= A$

54. If A is a skew symmetric matrix, then A^{2021} is

(A) Skew symmetric matrix (B) Row matrix

(C) Column matrix (D) Symmetric matrix

Ans (A)

Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$$A^3 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 8 & 0 \end{bmatrix}$$

$A^{2021} = A^{odd}$ is skew symmetric matrix.

55. If $f(1) = 1, f'(1) = 3$ then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(A) 12 (B) 10 (C) 33 (D) 35

Ans (C)

$f(1) = 1, f'(1) = 3$

Derivative of $f(f(f(x))) + (f(x))^2$

$= f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) + 2f(x) \cdot f'(x)$

At $x = 1$

$= f'(f(1)) \cdot f'(1) \cdot f'(1) + 2f(1) \cdot f'(1)$

$= (3)(3)(3) + 2(1)(3) = 33$

56. If $y = x^{\sin x} + (\sin x)^x$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

(A) $\frac{\pi^2}{2}$ (B) $\frac{4}{\pi}$ (C) $\pi \log \frac{\pi}{2}$ (D) 1

Ans (D)

$y = x^{\sin x} + (\sin x)^x$

Let $u = x^{\sin x}$

$\log u = \sin x \cdot \log x$

$\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right]$ ...(1)

$v = (\sin x)^x$

$\log v = x \log(\sin x)$

$\frac{dv}{dx} = (\sin x)^x\left[\frac{x}{\sin x}\cos x + \log \sin x\right]$ ...(2)

Eqn (1) + Eqn (2) at $x = \frac{\pi}{2}$

$\frac{dy}{dx} = \frac{\pi}{2}\left[\frac{2}{\pi}\right] + 1 \times (0)$

$= 1$

57. If $A_n \begin{bmatrix} 1-n & n \\ n & 1-n \end{bmatrix}$ then $|A_1| + |A_2| + \dots + |A_{2021}| =$

(A) 4042 (B) –2021 (C) $-(2021)^2$ (D) $(2021)^2$

Ans (C)

$|A_n| = (1-n)^2 - n^2$

$= 1 - 2n$

$|A_1| = -1$

$|A_2| = -3$

$|A_3| = -5$

$\therefore |A_1| + |A_2| + \dots + |A_{2021}|$

$= -1 - 3 - 5 \dots\dots (2021)$ terms

$= -(1 + 3 + 5 + \dots\dots 2021$ terms$)$

$= -(2021)^2$

58. If $y = (1 + x^2)\tan^{-1} x - x$ then $\frac{dy}{dx}$ is

(A) $x \tan^{-1} x$ (B) $2x \tan^{-1} x$ (C) $\frac{\tan^{-1} x}{x}$ (D) $x^2 \tan^{-1} x$

Ans (B)

$y = (1 + x^2)\tan^{-1} x - x$

$\frac{dy}{dx} = \frac{1+x^2}{1+x^2} + \tan^{-1} x(2x) - 1$

$= 2x \tan^{-1} x$

59. If $x = e^{\theta}\sin\theta$, $y = e^{\theta}\cos\theta$ where θ is a parameter, then $\frac{dy}{dx}$ at (1, 1) is equal to

(A) $-\frac{1}{4}$ (B) 0 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Ans (B)

$\frac{dy}{d\theta} = -e^{\theta}\sin\theta + \cos\theta e^{\theta}$

$\frac{dx}{d\theta} = e^{\theta}\cos\theta + \sin\theta e^{\theta}$

At (1, 1)

$e^{\theta}\sin\theta = e^{\theta}\cos\theta \therefore \theta = \pi/4$

$\frac{dy}{dx} = \frac{-\sin\theta + \cos\theta}{\cos\theta + \sin\theta}$

$\therefore \frac{dy}{dx} = 0$

60. If $y = e^{\sqrt{x\sqrt{x\sqrt{x}}}\ldots}$ $x > 1$ then $\frac{d^2y}{dx^2}$ at $x = \log_e^3$ is

(A) 1 (B) 3 (C) 5 (D) 0

Ans (B)

$y = e^{\sqrt{x\sqrt{x\sqrt{x}}}\ldots} = e^{x^{\frac{1}{2}}\cdot x^{\frac{1}{4}}\cdot x^{\frac{1}{8}}\ldots} = e^{x^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\ldots\right)}} = e^{x}$

$\frac{d^2y}{dx^2} = e^{x}$

$\left(\frac{d^2y}{dx^2}\right)_{x=\log_e^3} = e^{\log_e^3} = 3$

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